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SOLUTION OF THE INTERNAL INVERSE PROBLEM FOR A BULK ANISOTROPIC BODY

L. A. Kozdoba and V. N. Mudrikov UDC 536.242

An approach to the problem of modeling temperature fields of bulk anisotropic bodies is developed and realized on the basis of the equivalence principle.

Composite materials and structures that are bulk bodies from a set of elements consisting from diverse materials start to be used more and more in recent years. Radio electronic apparatus (REA) that must be considered as a large system are a typical example of such structures. The thermal diagrams of the functional REA subsystems are objects requiring a systemic and hierarchical approach. These methodological modes of solving thermal problems when using a whole series of principles of the phenomenological theory of heat and mass transfer permit optimization of the thermal regimes of the REA themselves and the thermal regimes of their production.

Thus, large systems consisting of many subsystems with sharply differentiated properties and numerous heat sources are the objects of mathematical modeling, computational experiment or methods and facilities of calculational thermophysics. The primary subsystems (homogeneous bulk bodies of complex shape), bulk and surface heat sources (sinks) produce large thermal systems for whose mathematical models of the thermal regimes there are no real values of the thermophysical characteristics, exact coordinates, and heat sources sufficiently well known in space and time.

One of the possible means of investigating the thermal regimes of such thermal systems is the experimental—theoretical approach. Underlying it are the effective (equivalent) values of the other characteristics in the condition of single-valuedness of the mathematical model (MM) of heat transport.

The equivalent (effective) values of the listed quantities can be obtained from solutions of the inverse problem [1]. Solutions of the direct problems (DP) and the optimal control problems (OCP) can be obtained by using such effective (equivalent) quantities or functions if the equivalence principle of the phenomenological theory of heat conduction is used [2]. Its crux is the following. As an example we consider the problem in which the thermophysical characteristics of an anisotropic structure will be equivalent. In substance, the mode of determining the thermophysical characteristics of an anisotropic body is elucidated when homogenization is performed. The homogenization principle has been known long and used successfully since the time when the MM of a homogeneous body is considered instead of the MM of the thermal regime of an anisotropic body.

The equivalence principle suggests just the means of a more or less exact determination of the values of the characteristics for the MM after homogenization. Let the temperature field of an anisotropic body be described by MM1. We determine the experimental temperatures T_e on a real object or on the model of an anisotropic body at certain points. The number of thermocouples and their disposition is not regulated.

The MM2 will be the model for the solution of the inverse problem (inverse, internal inverse, coefficient). This model of the thermal regime is written for a homogeneous body. We find the constant thermophysical characteristics by any method of solving the inverse

Institute of Technical Thermophysics, Academy of Sciences of the Ukrainian SSR, Kiev.
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problems [1]. Their values minimize the functional of the residuals $\Phi(\varepsilon)$, where $\varepsilon = T_m - T_e$. The minimization method and the form of the functional are determined by the investigator. The solution of the question of the necessity of applying regularization depends on it. The main thing that the characteristics found permit is to obtain the temperature field by solving the DP, by some criterion $\Phi(\varepsilon)$ closest to the (equivalent) field that should have been obtained by using the MM2 for an anisotropic body. Here T_e are the temperatures obtained by experiment on a physical object whose temperature field is described by the MM1.

This principle is utilized in linearization, in reduction (diminution of the dimensionality), in deformation, etc., i.e., always when the characteristics are obtained by starting from another (ordinarily simpler) MM1. But the experimental temperatures are obtained for the case when the MM is known to be more complex than the MM1. The terminology "equivalence" is used because the temperature fields obtained from the MM1 and MM2 are equivalent in the criterion selected.

An REA module is examined below for which the thermophysical characteristics of its subsystems were, with a small exception, unknown. Thermocouples were installed on the module model, at 7-10 points inside and outside the physical model. Nonstationary temperature fields were obtained at several heating and cooling regimes, with and without sources. These fields were initial for obtaining the equivalent heat conduction coefficients λ_{ef} and specific volume heats c_{vef}. Let us recall that the DP has a unique solution, consequently, the temperature for an anisotropic body with or without sources can never agree with the field for a homogeneous body. But these fields can be sufficiently near each other at certain points and as the temperature changes in time. If the constructions of real anisotropic modules (as their physical models also) are sufficiently close to each other in the size, location and number of elements, in the location and powers of the sources the found λ_{ef} and c_{vef} will yield sufficiently close temperature fields to the real ones in the DP and OCP. In the general case the temperature fields were different by not more than ±6% from the maximal temperatures.

Inverse (internal inverse) problems were solved on hybrid and digital computers. The analog portion in hybrid machines was a network processor, a network of variable resistors (R—R-network). A hybrid computer (HC) permitted automation of the known method of solving inverse problems described in the monograph [3]. Two- and three-dimensional problems were solved on the digital computer ES-1045 for those cases when the hybrid HC processor had an insufficient number of nodes of the nonuniform mesh for the solution of the DP by finite differences. As also follows from theoretical considerations, a change in the number and location of the thermocouples and a change in the number and power of the sources would result in a change in the values of λ_{ef} and c_{vef}. However, the errors did not exceed the maximally achievable (±10%) in any of the control examples. The equivalence principle was utilized even for the case when the equivalent characteristics were determined by starting from the MM of the thermal state of the body with orthotropic characteristics. It could be assumed that placement of the plates in a cylindrical module will result in orthotropy of $\lambda(\lambda_r \neq \lambda_z)$ (see Fig. 1). The solution of the control DP showed that even in this case λ_{ef} and c_{vef} obtained by starting from the model of a homogeneous module could be used.

Let us consider that the criterion in which the maximal absolute values of the residuals $\varepsilon = T_m - T_e$ are contained is the most reliable from the viewpoint of future computations of the maximal temperatures. The REA reliability depends on the maximal values of the temperatures. The functional containing the sum of the residuals, or a quadratic functional with sums over the coordinates, leads in time to the determination of λ_{ef} and c_{vef} yielding good integral but poor local temperature field patterns. The search for λ and c_v by the minimax criterion (Chebyshev approximation) complicates the search procedure [4] but yields more reliable (from the viewpoint of dangerous temperatures) representations of the thermal regimes.

The methodologies of the solution on HC and the programs for solution of inverse problems on a digital computer permit utilization of any criterion, consequently, the selection of the kind of criterion depends on the a priori conditions in the formulation of the problem.

The systemic and hierarchical approaches afforded a possibility of selecting those REA thermal regimes and those mathematical models of the thermal regimes that would permit obtaining equivalent characteristics satisfactory in accuracy for the solution of the OCP un-

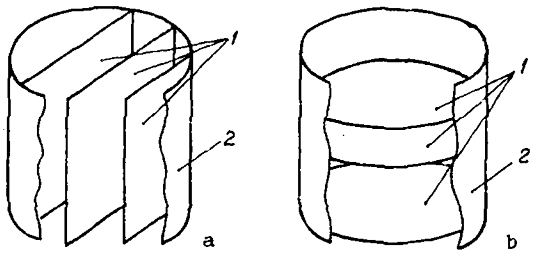

Fig. 1. Plate arrangement in a REA module [a) longitudinal, and b) transverse]; 1) plates, and 2) module housing.

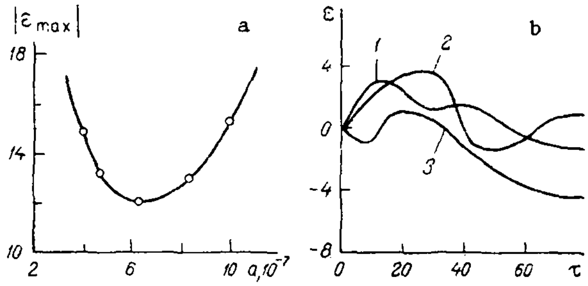

Fig. 2. Residual $\varepsilon = T_m - T_e$: a) solution of the inverse problem; b) control problem; 1) at the radius $12 \cdot 10^{-3}$ m; 2) $31 \cdot 10^{-3}$ m; 3) $47 \cdot 10^{-3}$ m. $|\varepsilon_{max}|$, ε, °C; a, m²/sec; τ, min.

der minimal expenditures of time and facilities. Two structural and thermal diagrams of cylindrical modules with plates disposed parallel and perpendicular to the axis of the cylindrical housing are displayed in Fig. 1. The experimental temperatures were taken off at six points on the plates and at one on the cylindrical housing surface. In principle the sites for thermocouple disposition were selected in a random manner so that sometimes the heads of the thermocouples were on heat liberating elements, sometimes simply on the plates, and sometimes in the air gap between the plates. Absolute values of the maximal residuals are shown in Fig. 2a for different magnitudes of the desired $a_{ef} = \lambda_{ef}/c_{vef}$. As is seen, minimization is performed comparatively simply, i.e., after 4-6 trial computations the minimum of the residuals is easily disclosed.

It is seen from Fig. 2b that the absolute values of the errors in the control example with the sources does not exceed 5 K.

In conclusion it must be noted that the equivalent values we obtained for the thermophysical characteristics ($\lambda_{ef} = 0.2$ W/(m·K)) agree satisfactorily with the data of other authors for analogous systems [5].

NOTATION

T_m, model temperature; T_e, experimental temperature; ε, residual; λ_{ef}, equivalent heat conduction coefficient; c_{vef}, equivalent volume specific heat.

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SOLUTION OF INVERSE PROBLEMS OF THE MECHANICS OF REACTIVE MEDIA

A. M. Grishin, A. Ya. Kuzin, S. P. Sinitsyn, and N. A. Yaroslavtsev

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The general inverse problem of heat and mass transfer in a porous reactive material is reduced to a set of particular problems by using splitting of the problem according to chemical and physical processes. A brief exposition is given of the methods for solving these particular inverse problems.

I. To raise the accuracy of computations in the mechanics of reactive media, more and more complex formulations of the problem are utilized at this time [1]. The complication proceeds in the direction of a more complete and detailed accounting of the structure and manifold of physicochemical transformations, extensive propagation of the conjugate as well as the two- and three-dimensional formulations of the problems. In all cases the extensive introduction into practice is repressed by the absence of information about the thermophysical coefficients, the thermokinetic constants of heterogeneous and homogeneous chemical reactions, and the flow characteristics on the surface of the streamlined reactive body. The characteristics mentioned can be determined as a result of solving inverse problems.

The following scheme is proposed for investigating complex inverse problems: 1) the most complete mathematical model is written on the basis of an analysis of a physicochemical model of the processes; 2) the most essential factors are exposed, say, on the basis of an analysis of dimensionalities and similarity, time expenditures of the researcher and the computer are taken into account and a more optimal, compromise mathematical model is constructed; 3) a search of available literature data is conducted, laboratory tests are planned and performed on specimens of the materials being investigated under conditions as close as possible to the full-scale conditions in order to determine the unknown characteristics and to obtain information to estimate the adequacy of the mathematical model; 4) appropriate particular inverse problems are posed and solved; 5) the direct problem is solved by using the obtained transfer coefficients and thermokinetic constants; the solution obtained is compared with full-scale and model experimental data.

As an example, let us consider the mathematical model of heat and mass transfer processes in a porous reactive material (glass-plastic) in a one-dimensional nonstationary formulation

$$\frac{\partial}{\partial t}(\rho_1\varphi_1) = -\rho_1\varphi_1 k_1 \exp\left(-\frac{E_1}{RT}\right), \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_2\varphi_2) = \alpha_1\rho_1\varphi_1 k_1 \exp\left(-\frac{E_1}{RT}\right) - \rho_2\varphi_2 k_2 \exp\left(-\frac{E_2}{RT}\right), \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_3\varphi_3) = \alpha_2\rho_2\varphi_2 k_2 \exp\left(-\frac{E_2}{RT}\right),\ \rho_4\varphi_4 = \text{const}, \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_5\varphi_5) + \frac{\partial}{\partial y}(\rho_5\varphi_5 v) = (1-\alpha_1)\rho_1\varphi_1 k_1 \exp\left(-\frac{E_1}{RT}\right) + (1-\alpha_2)\rho_2\varphi_2 k_2 \exp\left(-\frac{E_2}{RT}\right), \tag{4}$$

$$\rho_5\varphi_5\left(\frac{\partial c_\alpha}{\partial t} + v\frac{\partial c_\alpha}{\partial y}\right) = \frac{\partial}{\partial y}\left(\rho_5\varphi_5 D_\alpha \frac{\partial c_\alpha}{\partial y}\right) - c_\alpha R_5 + R_{5\alpha},\ \alpha = 1,\ 2, \tag{5}$$

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